Exam #1

Processamento de Imagem e Visão, 2008/09

1. *[color]* Color is an important feature in many image analysis problems.
   (a) explain color perception;
   (b) how is color synthesized in a screen?
   (c) how can we represent objects with multiple colors in images (e.g., people)?

2. *[image alignment]* We wish to align a pair of images using two sets of corresponding points. Let $(\mathbf{x}^1, \mathbf{y}^1), \ldots, (\mathbf{x}^K, \mathbf{y}^K)$, be K pairs of points, related by a geometric transform

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{e} \qquad \mathbf{x}, \mathbf{y}, \mathbf{e} \in \mathbb{R}^{\mathbf{2}} \qquad \mathbf{A} = \begin{bmatrix} 0 & a \\ b & c \end{bmatrix}$$

   where a, b, c are coefficients to be estimated and $\mathbf{e}$ is an alignment error. Estimate the unknown coefficients a, b, c by minimizing the squared error criterion

$$E = \sum_{i=1}^{K} \|\mathbf{y}^i - \mathbf{A}\mathbf{x}^i\|^2$$

3. *[interpolation]* Consider a finite length signal $\mathbf{x} = [x_1, \ldots, x_N]^T$, $(N = 8)$ and suppose we know some of its samples at specific time instants: $y_n = x_n$, for $n \in S = \{1, 3, 7\}$ and $y_1 = 1, y_3 = 3; y_7 = -1$.
   (a) Estimate the the unknown samples of vector $\mathbf{x}$ by minimizing the smoothness criterion

$$\mathcal{S} = \sum_{n=2}^{N} (x_n - x_{n-1})^2 ,$$

   (write the method and solution using matrix notation);
   (b) assume now that the observed samples are corrupted by additive noise $y_n = x_n + w_n, n \in S$ where w_n is a realization of a white noise process with zero mean. Estimate the vector $\mathbf{x}$ in this case.

4. *[edge detection]* Choose a method for edge detection in images and describe it. Each step of the algorithm should be mathematically defined.

5. *[camera model]* Consider a projective camera described by the model $\lambda\tilde{\mathbf{x}} = \mathbf{P}\tilde{\mathbf{X}}$ where $\mathbf{P}$ is a 3×4 projective matrix and $\tilde{\mathbf{X}}, \tilde{\mathbf{x}}$ are the vectors representing a point P in space and its projection on the camera plane in homogeneous coordinates.
   (a) Given a set of parallel lines in space $\mathbf{X} = \mathbf{X}_i + \alpha\mathbf{V}, i = 1, \ldots, N$, show that each projected line has a vanishing point and all the vanishing points are equal.
   (b) Can we calibrate the camera (i.e., estimate matrix $\mathbf{P}$) if we know several sets of parallel lines and their vanishing points? why?